

Name: _____

Date: _____

PC: Decomposition of Functions

Do Now:

Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$, find

(a) $f(g(4))$

(b) $f(g(x))$

A composite function is a function that brings together two or more functions. For instance, let h be given by

$$h(x) = \sqrt{x^2 + 2x + 2}$$

If we let $f(x) = x^2 + 2x + 2$ and $g(x) = \sqrt{x}$, then $(g \circ f)(x) =$

Thus the given function h has been *decomposed* into the composition of the two functions f and g . Such decompositions are not unique. More than one decomposition is possible.

We could have decomposed h into $f(x) = \sqrt{x+2}$ and $g(x) = x^2 + 2x$.

$$f(g(x)) =$$

We are going to avoid using the identity function ($f(x) = x$) in our decompositions.

1. Find the functions f and g so that $h(x) = f(g(x))$

(a) $h(x) = (3x+1)^2$

(b) $h(x) = \sqrt{1-4x}$

(c) $h(x) = \sqrt[4]{x+9}$

Practice

Express the function in the form $f \circ g$

1. $F(x) = (x - 9)^5$

4. $F(x) = \frac{1}{x+3}$

2. $F(x) = \sqrt{x} + 1$

5. $F(x) = |1 - x^3|$

3. $F(x) = \frac{x^2}{x^2 + 4}$

6. $F(x) = \sqrt{1 + \sqrt{x}}$

Express the function in the form $f \circ g \circ h$

7. $F(x) = \frac{1}{x^2 + 1}$

8. Express each of the following below as composites of two or more of the following:

$$a(x) = x + 1 \quad g(x) = x^3 \quad b(x) = x - 2 \quad h(x) = \frac{1}{x}$$

$$e(x) = 3x \quad k(x) = \sqrt{x} \quad f(x) = x^2$$

(a) $3x + 1$

(h) $x + 2$

(o) $\frac{1}{\sqrt{x}}$

(b) $3x + 3$

(i) $x - 1$

(p) $\frac{1}{\sqrt{x-2}}$

(c) $3x^2$

(j) $x^2 - 1$

(q) $\frac{1}{\sqrt{x-2}}$

(d) $9x^2$

(k) $3x + 2$

(r) $x^{\frac{3}{2}}$

(e) $(x^3 - 2)^2$

(l) $\sqrt{x^3 + 1}$

(s) $\frac{1}{3}x$

(f) $9x + 3$

(m) $\sqrt{x+1}$

(g) $\frac{1}{\sqrt{x^2+1}}$

(n) $\sqrt{x} + 1$